CHAPTER 8
Momentum

Figure 8.1 NFC defensive backs Ronde Barber and Roy Williams along with linebacker Jeremiah Trotter gang tackle AFC running back LaDainian Tomlinson during the 2006 Pro Bowl in Hawaii. (United States Marine Corps)

Chapter Outline

INTRODUCTION We know from everyday use of the word *momentum* that it is a tendency to continue on course in the same direction. Newscasters speak of sports teams or politicians gaining, losing, or maintaining the momentum to win. As we learned when studying about inertia, which is Newton's first law of motion, every object or system has inertia—that is, a tendency for an object in motion to remain in motion or an object at rest to remain at rest. Mass is a useful variable that lets us quantify inertia. Momentum is mass in motion.

Momentum is important because it is conserved in isolated systems; this fact is convenient for solving problems where objects collide. The magnitude of momentum grows with greater mass and/or speed. For example, look at the football players in the photograph (Figure 8.1). They collide and fall to the ground. During their collisions, momentum will play a large part. In this chapter, we will learn about momentum, the different types of collisions, and how to use momentum equations to solve collision problems.

8.1 Linear Momentum, Force, and Impulse

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe momentum, what can change momentum, impulse, and the impulse-momentum theorem
- Describe Newton's second law in terms of momentum
- Solve problems using the impulse-momentum theorem

Section Key Terms

change in momentum	impulse	impulse–momentum theorem	linear momentum

Momentum, Impulse, and the Impulse-Momentum Theorem

Linear momentum is the product of a system's mass and its velocity. In equation form, linear momentum $\mathbf{p}$ is

$$\mathbf{p} = m\mathbf{v}.$$

You can see from the equation that momentum is directly proportional to the object's mass (m) and velocity ($\mathbf{v}$). Therefore, the greater an object's mass or the greater its velocity, the greater its momentum. A large, fast-moving object has greater momentum than a smaller, slower object.

Momentum is a vector and has the same direction as velocity $\mathbf{v}$. Since mass is a scalar, when velocity is in a negative direction (i.e., opposite the direction of motion), the momentum will also be in a negative direction; and when velocity is in a positive direction, momentum will likewise be in a positive direction. The SI unit for momentum is kg m/s.

Momentum is so important for understanding motion that it was called the *quantity of motion* by physicists such as Newton. Force influences momentum, and we can rearrange Newton's second law of motion to show the relationship between force and momentum.

Recall our study of Newton's second law of motion ($\mathbf{F}_{\text{net}} = m\mathbf{a}$). Newton actually stated his second law of motion in terms of momentum: The net external force equals the **change in momentum** of a system divided by the time over which it changes. The change in momentum is the difference between the final and initial values of momentum.

In equation form, this law is

$$\mathbf{F}_{\text{net}} = \frac{\Delta \mathbf{p}}{\Delta t},$$

where $\mathbf{F}_{\text{net}}$ is the net external force, $\Delta \mathbf{p}$ is the change in momentum, and Δt is the change in time.

We can solve for $\Delta \mathbf{p}$ by rearranging the equation

$$\mathbf{F}_{\text{net}} = \frac{\Delta \mathbf{p}}{\Delta t}$$

to be

$$\Delta \mathbf{p} = \mathbf{F}_{\text{net}} \Delta t.$$

$\mathbf{F}_{\text{net}} \Delta t$ is known as **impulse** and this equation is known as the **impulse-momentum theorem**. From the equation, we see that the impulse equals the average net external force multiplied by the time this force acts. It is equal to the change in momentum. *The effect of a force on an object depends on how long it acts, as well as the strength of the force.* Impulse is a useful concept because it quantifies the effect of a force. A very large force acting for a short time can have a great effect on the momentum of an object, such as the force of a racket hitting a tennis ball. A small force could cause the same change in momentum, but it would have to act for a much longer time.

Newton's Second Law in Terms of Momentum

When Newton's second law is expressed in terms of momentum, it can be used for solving problems where mass varies, since $\Delta \mathbf{p} = \Delta(m\mathbf{v})$. In the more traditional form of the law that you are used to working with, mass is assumed to be constant. In fact, this traditional form is a special case of the law, where mass is constant. $\mathbf{F}_{\text{net}} = m\mathbf{a}$ is actually derived from the equation:

$$\mathbf{F}_{\text{net}} = \frac{\Delta\mathbf{p}}{\Delta t}$$

For the sake of understanding the relationship between Newton's second law in its two forms, let's recreate the derivation of $\mathbf{F}_{\text{net}} = m\mathbf{a}$ from

$$\mathbf{F}_{\text{net}} = \frac{\Delta\mathbf{p}}{\Delta t}$$

by substituting the definitions of acceleration and momentum.

The change in momentum $\Delta\mathbf{p}$ is given by

$$\Delta\mathbf{p} = \Delta(m\mathbf{v}).$$

If the mass of the system is constant, then

$$\Delta(m\mathbf{v}) = m\Delta\mathbf{v}.$$

By substituting $m\Delta\mathbf{v}$ for $\Delta\mathbf{p}$, Newton's second law of motion becomes

$$\mathbf{F}_{\text{net}} = \frac{\Delta\mathbf{p}}{\Delta t} = \frac{m\Delta\mathbf{v}}{\Delta t}$$

for a constant mass.

Because

$$\frac{\Delta\mathbf{v}}{\Delta t} = \mathbf{a},$$

we can substitute to get the familiar equation

$$\mathbf{F}_{\text{net}} = m\mathbf{a}$$

when the mass of the system is constant.

TIPS FOR SUCCESS

We just showed how $\mathbf{F}_{\text{net}} = m\mathbf{a}$ applies only when the mass of the system is constant. An example of when this formula would not apply would be a moving rocket that burns enough fuel to significantly change the mass of the rocket. In this case, you would need to use Newton's second law expressed in terms of momentum to account for the changing mass.

Snap Lab

Hand Movement and Impulse

In this activity you will experiment with different types of hand motions to gain an intuitive understanding of the relationship between force, time, and impulse.

- one ball
- one tub filled with water

Procedure:

1. Try catching a ball while *giving* with the ball, pulling your hands toward your body.
2. Next, try catching a ball while keeping your hands still.
3. Hit water in a tub with your full palm. Your full palm represents a swimmer doing a belly flop.
4. After the water has settled, hit the water again by diving your hand with your fingers first into the water. Your diving hand represents a swimmer doing a dive.
5. Explain what happens in each case and why.

GRASP CHECK

What are some other examples of motions that impulse affects?

a. a football player colliding with another, or a car moving at a constant velocity
b. a car moving at a constant velocity, or an object moving in the projectile motion
c. a car moving at a constant velocity, or a racket hitting a ball
d. a football player colliding with another, or a racket hitting a ball

LINKS TO PHYSICS

Engineering: Saving Lives Using the Concept of Impulse

Cars during the past several decades have gotten much safer. Seat belts play a major role in automobile safety by preventing people from flying into the windshield in the event of a crash. Other safety features, such as airbags, are less visible or obvious, but are also effective at making auto crashes less deadly (see Figure 8.2). Many of these safety features make use of the concept of impulse from physics. Recall that impulse is the net force multiplied by the duration of time of the impact. This was expressed mathematically as $\Delta \mathbf{p} = \mathbf{F}_{\text{net}} \Delta t$.

Figure 8.2 Vehicles have safety features like airbags and seat belts installed.

Airbags allow the net force on the occupants in the car to act over a much longer time when there is a sudden stop. The momentum change is the same for an occupant whether an airbag is deployed or not. But the force that brings the occupant to a stop will be much less if it acts over a larger time. By rearranging the equation for impulse to solve for force $\mathbf{F}_{\text{net}} = \frac{\Delta \mathbf{p}}{\Delta t}$, you can see how increasing Δt while $\Delta \mathbf{p}$ stays the same will decrease $\mathbf{F}_{\text{net}}$. This is another example of an inverse relationship. Similarly, a padded dashboard increases the time over which the force of impact acts, thereby reducing the force of impact.

Cars today have many plastic components. One advantage of plastics is their lighter weight, which results in better gas mileage. Another advantage is that a car will crumple in a collision, especially in the event of a head-on collision. A longer collision time means the force on the occupants of the car will be less. Deaths during car races decreased dramatically when the rigid frames of racing cars were replaced with parts that could crumple or collapse in the event of an accident.

GRASP CHECK

You may have heard the advice to bend your knees when jumping. In this example, a friend dares you to jump off of a park bench onto the ground without bending your knees. You, of course, refuse. Explain to your friend why this would be a foolish thing. Show it using the impulse-momentum theorem.

a. Bending your knees increases the time of the impact, thus decreasing the force.
b. Bending your knees decreases the time of the impact, thus decreasing the force.
c. Bending your knees increases the time of the impact, thus increasing the force.
d. Bending your knees decreases the time of the impact, thus increasing the force.

Solving Problems Using the Impulse-Momentum Theorem

WORKED EXAMPLE

Calculating Momentum: A Football Player and a Football

(a) Calculate the momentum of a 110 kg football player running at 8 m/s. (b) Compare the player's momentum with the momentum of a 0.410 kg football thrown hard at a speed of 25 m/s.

Strategy

No information is given about the direction of the football player or the football, so we can calculate only the magnitude of the momentum, *p*. (A symbol in italics represents magnitude.) In both parts of this example, the magnitude of momentum can be calculated directly from the definition of momentum:

$$\mathbf{p} = m\mathbf{v}$$

Solution for (a)

To find the player's momentum, substitute the known values for the player's mass and speed into the equation.

$$\mathbf{p}_{\text{player}} = (110 \text{ kg})(8 \text{ m/s}) = 880 \text{ kg} \cdot \text{m/s}$$

Solution for (b)

To find the ball's momentum, substitute the known values for the ball's mass and speed into the equation.

$$\mathbf{p}_{\text{ball}} = (0.410 \text{ kg})(25 \text{ m/s}) = 10.25 \text{ kg} \cdot \text{m/s}$$

The ratio of the player's momentum to the ball's momentum is

$$\frac{\mathbf{p}_{\text{player}}}{\mathbf{p}_{\text{ball}}} = \frac{880}{10.3} = 85.9 \text{ .}$$

Discussion

Although the ball has greater velocity, the player has a much greater mass. Therefore, the momentum of the player is about 86 times greater than the momentum of the football.

WORKED EXAMPLE

Calculating Force: Venus Williams' Racquet

During the 2007 French Open, Venus Williams (Figure 8.3) hit the fastest recorded serve in a premier women's match, reaching a speed of 58 m/s (209 km/h). What was the average force exerted on the 0.057 kg tennis ball by Williams' racquet? Assume that the ball's speed just after impact was 58 m/s, the horizontal velocity before impact is negligible, and that the ball remained in contact with the racquet for 5 ms (milliseconds).

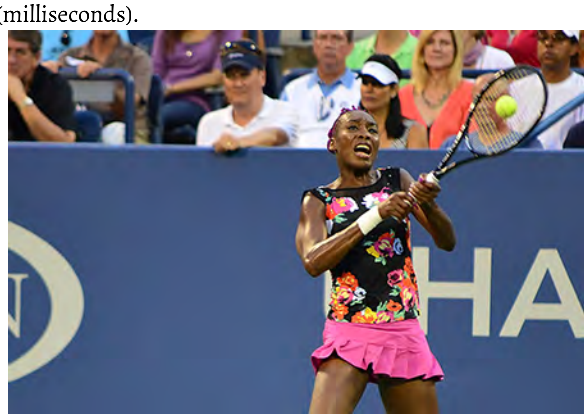

Figure 8.3 Venus Williams playing in the 2013 US Open (Edwin Martinez, Flickr)

Strategy

Recall that Newton's second law stated in terms of momentum is

$$\mathbf{F}_{\text{net}} = \frac{\Delta \mathbf{p}}{\Delta t}.$$

As noted above, when mass is constant, the change in momentum is given by

$$\Delta \mathbf{p} = m\Delta \mathbf{v} = m(\mathbf{v}_{\text{f}} - \mathbf{v}_{\text{i}}),$$

where $\mathbf{v}_{\text{f}}$ is the final velocity and $\mathbf{v}_{\text{i}}$ is the initial velocity. In this example, the velocity just after impact and the change in time are given, so after we solve for $\Delta \mathbf{p}$, we can use $\mathbf{F}_{\text{net}} = \frac{\Delta \mathbf{p}}{\Delta t}$ to find the force.

Solution

To determine the change in momentum, substitute the values for mass and the initial and final velocities into the equation above.

$$\begin{aligned} \Delta \mathbf{p} &= m(\mathbf{v}_f - \mathbf{v}_i) \\ &= (0.057 \text{ kg})\,(58 \text{ m/s} - 0 \text{ m/s}) \\ &= 3.306 \text{ kg·m/s} \approx 3.3 \text{ kg·m/s} \end{aligned} \tag{8.1}$$

Now we can find the magnitude of the net external force using $\mathbf{F}_{\text{net}} = \frac{\Delta \mathbf{p}}{\Delta t}$

$$\begin{aligned} \mathbf{F}_{\text{net}} &= \frac{\Delta \mathbf{p}}{\Delta t} = \frac{3.306}{5 \times 10^{-3}} \\ &= 661 \text{ N} \\ &\approx 660 \text{ N}. \end{aligned} \tag{8.2}$$

Discussion

This quantity was the average force exerted by Venus Williams' racquet on the tennis ball during its brief impact. This problem could also be solved by first finding the acceleration and then using $\mathbf{F}_{\text{net}} = m\mathbf{a}$, but we would have had to do one more step. In this case, using momentum was a shortcut.

Practice Problems

1. What is the momentum of a bowling ball with mass 5 kg and velocity 10 m/s?
   a. $0.5 \text{ kg} \cdot \text{m/s}$
   b. $2 \text{ kg} \cdot \text{m/s}$
   c. $15 \text{ kg} \cdot \text{m/s}$
   d. $50 \text{ kg} \cdot \text{m/s}$

2. What will be the change in momentum caused by a net force of 120 N acting on an object for 2 seconds?
   a. $60 \text{ kg} \cdot \text{m/s}$
   b. $118 \text{ kg} \cdot \text{m/s}$
   c. $122 \text{ kg} \cdot \text{m/s}$
   d. $240 \text{ kg} \cdot \text{m/s}$

Check Your Understanding

3. What is linear momentum?
   a. the sum of a system's mass and its velocity
   b. the ratio of a system's mass to its velocity
   c. the product of a system's mass and its velocity
   d. the product of a system's moment of inertia and its velocity

4. If an object's mass is constant, what is its momentum proportional to?
   a. Its velocity
   b. Its weight
   c. Its displacement
   d. Its moment of inertia

5. What is the equation for Newton's second law of motion, in terms of mass, velocity, and time, when the mass of the system is

constant?

a. $\mathbf{F}_{\text{net}} = \frac{\Delta \mathbf{v}}{\Delta m \Delta t}$
b. $\mathbf{F}_{\text{net}} = \frac{m \Delta t}{\Delta \mathbf{v}}$
c. $\mathbf{F}_{\text{net}} = \frac{m \Delta \mathbf{v}}{\Delta t}$
d. $\mathbf{F}_{\text{net}} = \frac{\Delta m \Delta \mathbf{v}}{\Delta t}$

6. Give an example of a system whose mass is not constant.
a. A spinning top
b. A baseball flying through the air
c. A rocket launched from Earth
d. A block sliding on a frictionless inclined plane

8.2 Conservation of Momentum

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe the law of conservation of momentum verbally and mathematically

Section Key Terms

angular momentum	isolated system	law of conservation of momentum

Conservation of Momentum

It is important we realize that momentum is conserved during collisions, explosions, and other events involving objects in motion. To say that a quantity is conserved means that it is constant throughout the event. In the case of conservation of momentum, the total momentum in the system remains the same before and after the collision.

You may have noticed that momentum was *not* conserved in some of the examples previously presented in this chapter. where forces acting on the objects produced large changes in momentum. Why is this? The systems of interest considered in those problems were not inclusive enough. If the systems were expanded to include more objects, then momentum would in fact be conserved in those sample problems. It is always possible to find a larger system where momentum is conserved, even though momentum changes for individual objects within the system.

For example, if a football player runs into the goalpost in the end zone, a force will cause him to bounce backward. His momentum is obviously greatly changed, and considering only the football player, we would find that momentum is not conserved. However, the system can be expanded to contain the entire Earth. Surprisingly, Earth also recoils—conserving momentum—because of the force applied to it through the goalpost. The effect on Earth is not noticeable because it is so much more massive than the player, but the effect is real.

Next, consider what happens if the masses of two colliding objects are more similar than the masses of a football player and Earth—in the example shown in Figure 8.4 of one car bumping into another. Both cars are coasting in the same direction when the lead car, labeled m_2, is bumped by the trailing car, labeled m_1. The only unbalanced force on each car is the force of the collision, assuming that the effects due to friction are negligible. Car *m*1 slows down as a result of the collision, losing some momentum, while car *m*2 speeds up and gains some momentum. If we choose the system to include both cars and assume that friction is negligible, then the momentum of the two-car system should remain constant. Now we will prove that the total momentum of the two-car system does in fact remain constant, and is therefore conserved.